

# Emergent eigenstate solution to quantum dynamics far from equilibrium

Lev Vidmar,<sup>1</sup> Deepak Iyer,<sup>2</sup> and Marcos Rigol<sup>1</sup>

<sup>1</sup>*Department of Physics, The Pennsylvania State University, University Park, PA 16802, USA*

<sup>2</sup>*Bucknell University, Lewisburg, PA 17837, USA*

Recent studies on the dynamics of interacting quantum many-body systems have shed light into this largely unexplored domain in physics [1, 2]. A unique possibility revealed by such studies is the dynamical realization of novel quantum states. Prominent examples are prethermal states close to integrability [3, 4] and topological states in periodically driven systems [5, 6]. The former can be understood as steady states of integrable Hamiltonians and the latter via effective Floquet descriptions. Here, we unveil another surprising outcome of quantum dynamics, the generation of time-evolving states that are eigenstates of emergent local Hamiltonians (not trivially related to the ones driving the dynamics). We study far from equilibrium dynamics of fermionic and bosonic systems in one-dimensional lattices, and provide examples of physically relevant time-evolving states that are either ground states or highly excited eigenstates of emergent Hamiltonians. We examine the site and momentum distributions, as well as the strikingly different correlations that emerge in such systems.

Impressive experiments with ultracold gases [3, 4], photonic [5], and solid state systems [6–8], and foundational theoretical developments [1, 2] are driving the study of out-of-equilibrium quantum many-body systems at a rapid pace. When taken far from equilibrium, generic isolated quantum systems typically thermalize [9], whereas integrable systems (characterized by an extensive number of local conserved quantities) do not. They are instead described by generalized Gibbs ensembles (GGEs) [10]. For nonintegrable systems close to integrable points, one expects relaxation to long-lived states (prethermal states [11]), that can also be described using GGEs [12]. Drawing from notions of the equilibrium renormalization group, prethermal states can be understood as being nonthermal fixed points [13]. More recently, the discovery of dynamical phase transitions [14], which are the result of nonanalytic behavior in time, has added another dimension to the connection between quantum dynamics and traditional statistical mechanics.

Here we add yet another paradigm to that already rich phenomenology. It is motivated by recent theoretical studies that have revealed the emergence of power-law correlations in various one-dimensional lattice systems of impenetrable [15] and soft-core [16] bosons, spinful fermions [17], and spins [18–20] far from equilibrium. Since one-dimensional systems in equilibrium generally exhibit power-law correlations only in the ground state, we propose, as a possible explanation for these findings, that the time-evolving states are ground states of *emergent local Hamiltonians*. More generally, we show that there are families of physically relevant time-evolving states that are eigenstates of such Hamiltonians. We first outline a framework within which this happens in quantum quenches and then discuss examples involving transport far from equilibrium. The latter has been a topic of much interest in recent experiments with ultracold gases in optical lattices [21–24].

In a quantum quench, the initial state  $|\psi_0\rangle$  is usually an eigenstate of some initial local Hamiltonian  $\hat{P}$ , such that  $(\hat{P} - \lambda)|\psi_0\rangle = 0$  ( $\lambda$  is the energy eigenvalue). At the time of the quench ( $t = 0$ ),  $\hat{P}$  is changed instantaneously into a new local Hamiltonian  $\hat{H}$ , after which the initial state evolves as  $|\psi(t)\rangle = e^{-i\hat{H}t}|\psi_0\rangle$  (we set  $\hbar = 1$  and assume that  $[\hat{H}, \hat{P}] \neq 0$ ). Inserting an identity  $\hat{I} = e^{i\hat{H}t}e^{-i\hat{H}t}$  in the initial eigenstate equation, and multiplying it by  $e^{-i\hat{H}t}$ , leads to:

$$(e^{-i\hat{H}t}\hat{P}e^{i\hat{H}t} - \lambda)|\psi(t)\rangle \equiv \hat{M}(t)|\psi(t)\rangle = 0, \quad (1)$$

where  $\hat{M}(t)$  is a time-dependent operator in the Schrödinger picture. In general,  $\hat{M}(t)$  is highly non-local and, hence, of no particular interest. Its non-local character is apparent in the expansion

$$\hat{M}(t) = \hat{P} - \lambda - it[\hat{H}, \hat{P}] + \frac{(it)^2}{2!}[\hat{H}, [\hat{H}, \hat{P}]] + \dots, \quad (2)$$

in which each higher-order commutator generally extends the spatial support of the products of operators involved in  $\hat{M}(t)$ . However, something remarkable occurs if

$$[\hat{H}, \hat{P}] = ia_0\hat{Q}, \quad (3)$$

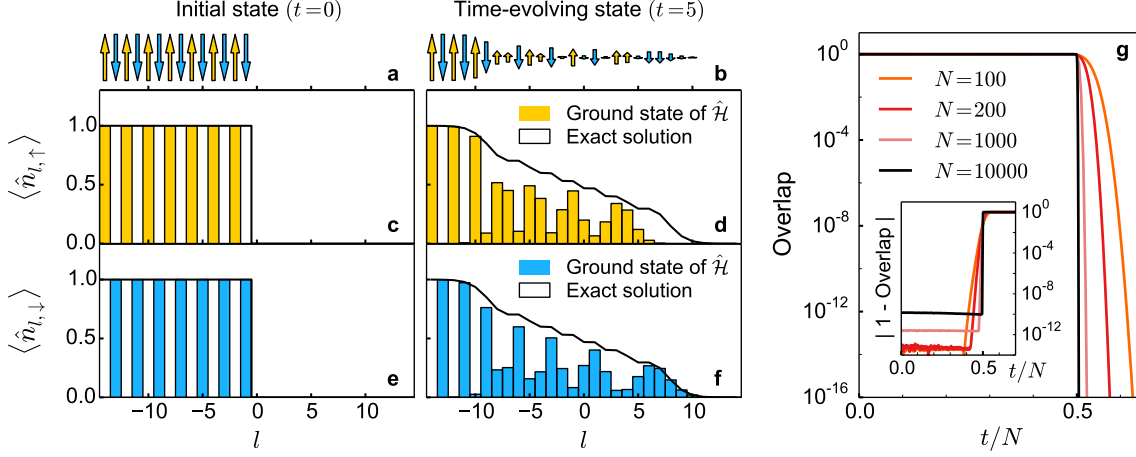


FIG. 1. **Emergent Hamiltonian description of the Néel-domain-wall melting in the infinite-repulsion Fermi-Hubbard model.** **a**, Initial state for  $N = 14$  with all sites with  $l < 0$  occupied and all sites with  $l \geq 0$  empty. **b**, Spin distribution at  $t = 5$  (in units of  $\hbar/J$ ). The length of the arrows is proportional to  $|\langle \hat{n}_{l,\uparrow} \rangle - \langle \hat{n}_{l,\downarrow} \rangle|$ . The expectation values are computed with the exact time-evolving wavefunction  $|\psi(t)\rangle$ , and with the ground state  $|\Psi_t\rangle$  of the emergent Hamiltonian (5) with  $\lambda = -N(N+1)/2$ , yielding perfect agreement. **c** and **d**, Site occupation of spin-up fermions. **e** and **f**, Site occupation of spin-down fermions. The thick solid lines in **c-f** depict the total site occupation  $\langle \hat{n}_{l,\uparrow} + \hat{n}_{l,\downarrow} \rangle$ . **g**, Overlap  $O(t) = |\langle \Psi_t | \psi(t) \rangle|$  as a function of rescaled time  $t/N$  for four system sizes, and  $|1 - O(t)|$  in the inset. When increasing number of particles  $N$ ,  $O(t) = 1$  within machine precision (see inset) until  $t/N = 1/2$ .

where  $\hat{Q}$  is a conserved quantity ( $[\hat{H}, \hat{Q}] = 0$ ) and  $a_0$  is some constant. Then, all higher-order terms in Eq. (2) vanish and  $\hat{M}$  can be thought of as being an emergent local Hamiltonian

$$\hat{\mathcal{H}}(t) \equiv \hat{P} + a_0 t \hat{Q} - \lambda, \quad (4)$$

of which the time-evolving state is an eigenstate. If  $|\psi(t)\rangle$  were the ground state of such an emergent Hamiltonian, then all the theoretical tools developed to understand the ground state of quantum systems could immediately be applied to understand a system far from equilibrium. As we discuss below, there are indeed experimentally relevant time-evolving states that are ground states of such emergent local Hamiltonians. Furthermore, we show that it is possible to relax the requirement that  $\hat{Q}$  is exactly conserved and still satisfy  $\hat{\mathcal{H}}(t)|\psi(t)\rangle = 0$  over long times.

In recent experiments [22, 24], the initial state was taken to be a product state  $|\psi_0\rangle = \prod_{l \in L_0} \hat{b}_l^\dagger |\emptyset\rangle$ , where  $\hat{b}_l^\dagger$  creates a boson on site  $l$  of a one-dimensional lattice. The dynamics was studied in the presence of very strong interactions (impenetrable bosons), for which transport is ballistic. We denote the set of initially occupied sites by  $L_0 \equiv \{l_1, l_2, \dots, l_N\}$ , where  $N$  is the total number of par-

ticles. One can see that  $|\psi_0\rangle$  is an eigenstate of the family of operators  $\hat{P} = \sum_l \beta_l \hat{b}_l^\dagger \hat{b}_l$ , with arbitrary coefficients  $\beta_l$ . For concreteness, we consider  $|\psi_0\rangle$  to be a domain wall, in which the first  $N$  sites ( $L_0 \equiv \{-N, -N+1, \dots, -1\}$ ) of the lattice with open boundaries are occupied, and there are  $N+1$  empty sites ( $l \in \{0, 1, \dots, N\}$ ).

Since impenetrable bosons can be mapped onto noninteracting spinless fermions [25], to derive an explicit expression for the emergent Hamiltonian we focus on the latter. The fermionic Hamiltonian can be written as  $\hat{H} = -J \sum_{l=-N}^{N-1} (\hat{f}_{l+1}^\dagger \hat{f}_l + \text{h.c.})$ , where we set the hopping amplitude  $J = 1$ . By setting  $\beta_l = l$  in the operator  $\hat{P} = \sum_l \beta_l \hat{n}_l$  (with  $\hat{n}_l = \hat{f}_l^\dagger \hat{f}_l$ ), one gets that in Eq. (3)  $a_0 = -1$  and  $\hat{Q} = \sum_{l=-N}^{N-1} (i \hat{f}_{l+1}^\dagger \hat{f}_l + \text{h.c.})$ . Our initial state is the ground state of  $\hat{P}$ , and  $\hat{Q}$  is nothing but the current operator in an open lattice. The emergent Hamiltonian then has the form

$$\hat{\mathcal{H}}(t) = \sum_{l=-N}^N l \hat{n}_l - t \sum_{l=-N}^{N-1} (i \hat{f}_{l+1}^\dagger \hat{f}_l + \text{h.c.}) - \lambda. \quad (5)$$

Contrary to the derivation leading to Eq. (4), in the experimentally relevant setup  $\hat{Q}$  is not exactly conserved (the system is finite and has open

boundaries),  $[\hat{H}, \hat{Q}] = -2i(\hat{n}_{-N} - \hat{n}_N)$ , i.e.,  $\hat{Q}$  is conserved up to boundary terms. Nevertheless, for our initial state, this merely results in a redefinition  $\lambda \rightarrow \lambda(t) = \lambda - t^2$  and a restriction on the times for which the emergent Hamiltonian description is valid:  $t \lesssim N/e$  for  $N \gg 1$  (see Methods). Since the maximal group velocity in the lattice is  $v_{\max} = 2$  (in units of  $Ja/\hbar$ ,  $a$  is the lattice spacing), a physical picture consistent with the time restriction is that the emergent Hamiltonian description (5) is valid so long as the expanding particles (holes) do not reach the edge of the lattice,  $t \lesssim N/v_{\max} = N/2$ . During that time, the dynamics is expected to be the same as that of the semi-infinite domain wall. For the latter, the current is exactly conserved and  $\hat{\mathcal{H}}(t)|\psi(t)\rangle = 0$  is satisfied at all times. One can check this explicitly (see Supplementary Information).

We test the validity of these results numerically for finite systems. As mentioned before, impenetrable bosons can be mapped onto spinless fermions. For any given spin ordering, the same is true for the infinite-repulsion Fermi-Hubbard model (spin exchange is not possible). We have used such a mapping to devise a computational approach to compute equilibrium and nonequilibrium properties of the latter model in polynomial time (see Methods). We will be interested in exploring the Néel domain-wall melting. Various related Fock states have already been implemented in experimental studies of quantum dynamics in two-component fermionic systems [21, 26].

Figures 1a-f show the spin and occupation profiles during the melting of a Néel domain wall as obtained using the exact time evolution of the initial state and the ground state of the emergent Hamiltonian. The results of the two approaches are indistinguishable. A more stringent accuracy test is provided by the overlap between the time-evolving wavefunction  $|\psi(t)\rangle$  and the ground state of the emergent Hamiltonian  $|\Psi_t\rangle$ ,  $O(t) = |\langle\Psi_t|\psi(t)\rangle|$ , which is shown in Fig. 1g as a function of the rescaled time  $t/N$  for various system sizes. It exhibits a plateau with  $O(t) = 1$  within machine precision (see inset), and starts decreasing when  $t/N$  approaches  $1/2$  (as advanced). Hence, as  $N$  increases both wavefunctions become identical during the melting of the domain wall.

Next we study the one-particle correlations and the momentum distribution function during the domain-wall melting, and contrast the results for the infinite-repulsion Fermi-Hubbard model with those for impenetrable bosons. This is of interest since an intimate relation between current-carrying states and power-law correlations has been observed for over the last 30 years [15–20, 27, 28]. Whereas

the dynamics of the site occupations is the same for the infinite-repulsion Fermi-Hubbard model and impenetrable bosons (because of the mapping to spinless fermions), nonlocal one-particle correlations  $\mathcal{C}(x)$  (see caption in Fig. 2) are starkly different. Initially  $\mathcal{C}(x \neq 0) = 0$ , which means that the non-local correlations in Fig. 2 have developed dynamically. Figures 2a and 2c show that the correlations in the Fermi-Hubbard model exhibit a Gaussian decay  $\mathcal{C}(x) = 0.39 \exp(-0.35x^2)$ , in sharp contrast with the results for impenetrable bosons [Figs 2b and 2d], which at long times exhibit a power-law decay  $\mathcal{C}(x) = 0.29/\sqrt{x}$  [19].

These results explain why power law correlations can emerge during the far-from-equilibrium dynamics of quantum systems: the time-evolving states can be ground states of emergent local Hamiltonians. They also show that non-power-law (Gaussian) correlations are compatible with ballistic currents, and that they exist in the ground state of emergent Hamiltonians. The different nature of the one-particle correlations in the systems studied here can be proved experimentally by means of time-of-flight measurements of the quasi-momentum distribution function  $n(q)$ . The insets in Figs 2c and 2d show  $n(q)$  at the same time for which one-particle correlations are depicted in Figs 2a and 2b. As a result of quasi-long-range order [ $\mathcal{C}(x) \propto 1/\sqrt{x}$ ], and in stark contrast to the fermionic  $n(q)$ , the  $n(q)$  of impenetrable bosons exhibits a sharp peak [15, 24].

As expected from the generality of the framework introduced earlier, the emergent local Hamiltonian description also applies to interacting systems that cannot be mapped onto noninteracting ones. To show this, we study the expansion of a domain wall of spinless fermions,  $|\psi_0\rangle = \prod_{l=-N+1}^0 \hat{f}_l^\dagger |\emptyset\rangle$ , in the presence of nearest-neighbor interactions of strength  $V$ ,  $\hat{H}_V = \sum_{l=-N+1}^{N-1} \hat{h}_l(V)$ , with  $\hat{h}_l(V) = -( \hat{f}_{l+1}^\dagger \hat{f}_l + \text{h.c.} ) + V(\hat{n}_l - 1/2)(\hat{n}_{l+1} - 1/2)$ . In this case, the emergent Hamiltonian reads  $\hat{\mathcal{H}}_V(t) = \hat{P}(V) + t\hat{Q}(V)$ , where  $\hat{P}(V) = \sum_{l=-N+1}^{N-1} \hat{h}_l(V)$  and

$$\begin{aligned} \hat{Q}(V) = \sum_{l=-N+1}^{N-2} \bigg\{ & (i\hat{f}_{l+2}^\dagger \hat{f}_l + \text{h.c.}) \\ & - V(i\hat{f}_{l+1}^\dagger \hat{f}_l + \text{h.c.})(\hat{n}_{l+2} - 1/2) \\ & - V(i\hat{f}_{l+2}^\dagger \hat{f}_{l+1} + \text{h.c.})(\hat{n}_l - 1/2) \bigg\}. \end{aligned} \quad (6)$$

$\hat{Q}(V)$  is known as the energy current operator and  $\hat{P}(V)$  as the boost operator [29]. The eigenstate  $|\Psi_t^V\rangle$  of  $\hat{\mathcal{H}}_V(t)$  that describes the dynamics is a highly excited state (see Methods).

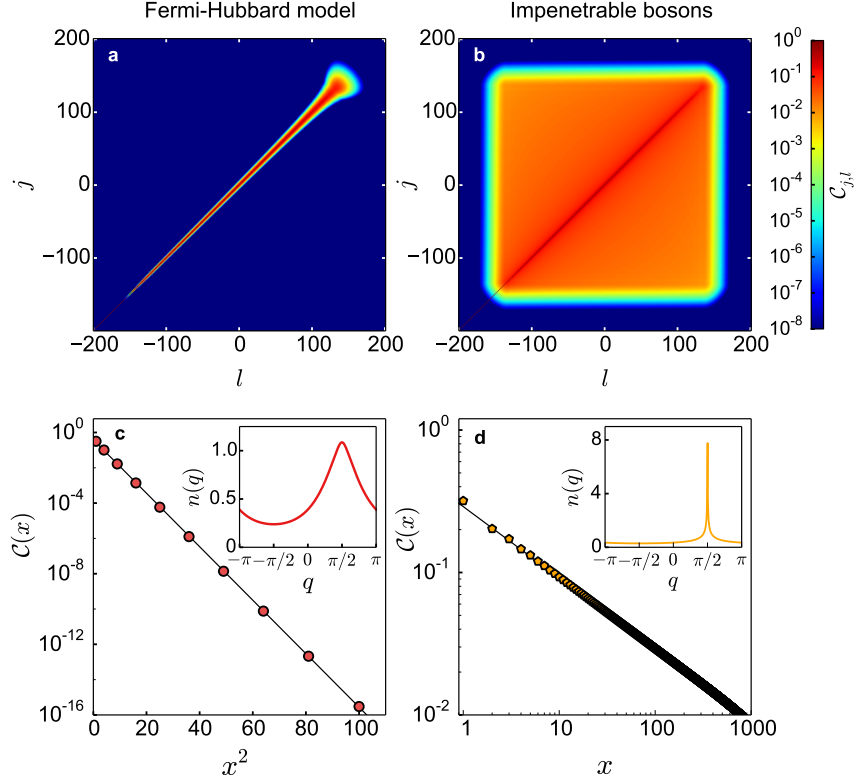
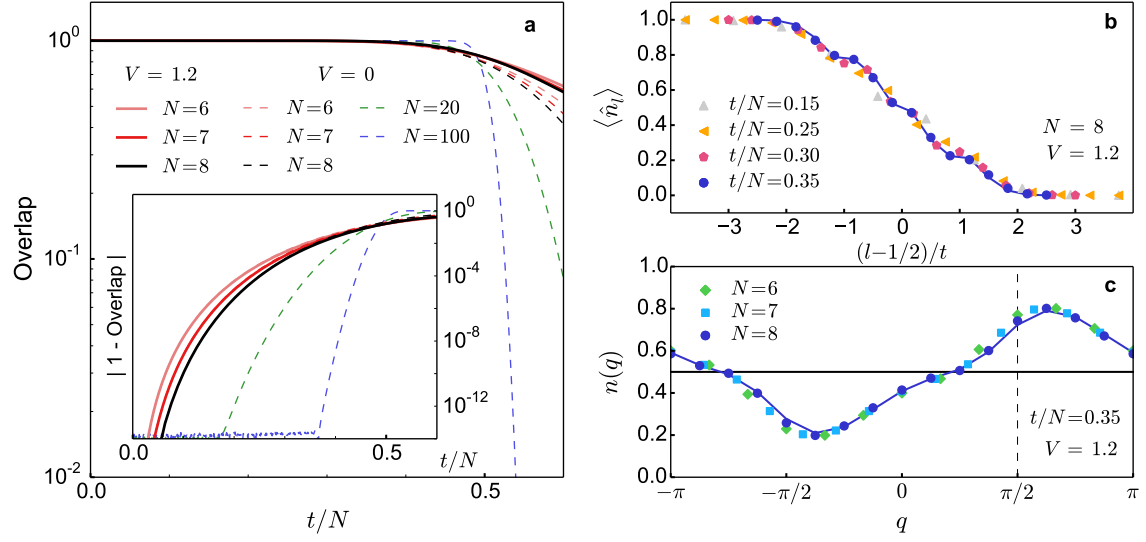


FIG. 2. **Distinct nature of nonlocal correlations in current-carrying states.** Quantum coherence is generated spontaneously as the domain walls melt. **a** and **b**, Absolute value of all elements of the one-particle density matrix for  $N = 200$  particles at time  $t/N = 0.35$ . **a**,  $C_{j,l} = |\langle \hat{f}_{j,\uparrow}^\dagger \hat{f}_{l,\uparrow} \rangle| + |\langle \hat{f}_{j,\downarrow}^\dagger \hat{f}_{l,\downarrow} \rangle|$  for the infinite-repulsion Fermi-Hubbard model. **b**,  $C_{j,l} = |\langle \hat{b}_j^\dagger \hat{b}_l \rangle|$  for impenetrable bosons. **c** and **d**,  $\mathcal{C}(x) \equiv \mathcal{C}_{0,x}$  as a function of  $x$  for the infinite-repulsion Fermi-Hubbard model (**c**) and for impenetrable bosons (**d**), calculated for  $N = 2000$  particles at time  $t/N = 0.35$ . The solid lines overlapping with the data are a fit to  $\mathcal{C}(x) = 0.39 \exp(-0.35x^2)$  for the infinite-repulsion Fermi-Hubbard model, and  $\mathcal{C}(x) = 0.29/\sqrt{x}$  for hard-core bosons [19]. Insets in **c** and **d**, Quasi-momentum distribution function  $n(q) = \sum_{j,l} e^{iq(j-l)} C_{j,l} / (2N + 1)$  corresponding to the results in **a** and **b** ( $N = 200$ ), respectively. Whereas impenetrable bosons undergo dynamical quasi-condensation [15], recently observed experimentally [24], the infinite-repulsion Fermi-Hubbard model exhibits a momentum distribution function without sharp features.

In Fig. 3a, we plot the overlap between  $|\Psi_t^V\rangle$  and the exact time-evolved wavefunction  $|\psi^V(t)\rangle = e^{-i\hat{H}_V t}|\psi_0\rangle$ , for  $V = 0, 1.2$  and different system sizes. All the overlaps are nearly one for  $t/N \lesssim 0.5$  (see also the inset), independently of whether the system is interacting ( $V = 1.2$ ) or not ( $V = 0$ ). With increasing  $N$ , the behavior of the overlaps is qualitatively similar to that in Fig. 1. For  $V = 1.2$ , the corresponding site and quasi-momentum occupations are depicted in Figs. 3b and 3c, respectively. While the site occupations do not differ significantly from the noninteracting case [24], the quasi-momentum occupations develop a peak away from  $q = \pi/2$ , in

clear contrast to the results in the insets in Fig. 2. These results open a new door to further studies of the effects of interactions in such systems without the need of solving for the exact dynamics [20].

The systems considered so far are integrable. However, since our framework only requires finding one conserved (or almost conserved) quantity  $\hat{Q}$ , we foresee applications to nonintegrable systems. The emergence of power-law correlations in the (nonintegrable) one-dimensional Bose-Hubbard model [16], lends support to this expectation. We plan to explore this and the quantum dynamics of fermionic Mott insulators [17] in the near future.



**FIG. 3. Emergent Hamiltonian description of the domain-wall melting for interacting spinless fermions.** **a**, Overlap  $O(t) = |\langle \Psi_t^V | \psi^V(t) \rangle|$  as a function of rescaled time  $t/N$  for  $V = 1.2$  (from full exact diagonalization) and  $V = 0$  (noninteracting system), and different system sizes. The inset depicts  $|1 - O(t)|$ . For  $N = 6, 7$ , and  $8$  (see inset), the results for interacting and noninteracting fermions are virtually indistinguishable at short times. Results for larger system sizes are only available for the noninteracting case. **b** and **c**, Site and quasi-momentum occupations for  $V = 1.2$  obtained from  $|\psi^V(t)\rangle$  (symbols) and from  $|\Psi_t^V\rangle$  (lines; data shown for  $N = 8$  at time  $t/N = 0.35$ ). **b**, Site occupations  $\langle \hat{n}_l \rangle$  for  $N = 8$  and different times as a function of the rescaled coordinate  $(l - 1/2)/t$ . The data collapses onto a line [30] as for noninteracting particles [24]. **c**, Quasi-momentum distribution function  $n(q)$  for different system sizes and the same rescaled time  $t/N = 0.35$ . There is again data collapse. As a result of the interactions, the peak in  $n(q)$  is shifted from  $q = \pi/2$  as seen in Fig. 2.

## METHODS

### A. Models mappable to noninteracting ones.

The Hamiltonian for the one-dimensional Fermi-Hubbard model at infinite onsite repulsion is given by  $\hat{H}_{\text{FHM}} = -\sum_{l=-N}^{N-1} \sum_{\sigma \in \{\uparrow, \downarrow\}} (\hat{f}_{l+1, \sigma}^\dagger \hat{f}_{l, \sigma} + \text{h.c.})$ , where  $\hat{f}_{l, \sigma}^\dagger$  creates a fermion with spin  $\sigma$  at site  $l$ . Infinite repulsion is enforced by the constraints  $\hat{f}_{l, \sigma}^\dagger \hat{f}_{l, -\sigma}^\dagger = \hat{f}_{l, \sigma}^\dagger \hat{f}_{l, -\sigma}^\dagger = 0$ . Similarly, the one-dimensional Bose-Hubbard model at infinite onsite repulsion (impenetrable bosons) is defined by  $\hat{H}_{\text{BHM}} = -\sum_{l=-N}^{N-1} (\hat{b}_{l+1}^\dagger \hat{b}_l + \text{h.c.})$  with the constraints  $(\hat{b}_l)^2 = (\hat{b}_l^\dagger)^2 = 0$ , where  $\hat{b}_l^\dagger$  is the boson creation operator at site  $l$ .

### B. Spinless fermions with nearest-neighbor interactions.

In one dimension, the Hamiltonian  $\hat{H}_V$  can be mapped onto the well known spin-1/2 XXZ chain [25]. The operator  $\hat{P}(V)$  is known as the boost operator for the energy current operator  $\hat{Q}(V)$ ,  $[\hat{H}_V, \hat{P}(V)] = i\hat{Q}(V)$ .  $\hat{Q}(V)$  is exactly conserved ( $[\hat{H}_V, \hat{Q}(V)] = 0$ ) for periodic boundary conditions, and it is sometimes denoted as  $\hat{Q}_3$  because it has support on three lattice sites.

### C. Construction of the emergent local Hamiltonian.

To analyze the accuracy of our construction if  $\hat{Q}$  is not exactly conserved,  $[\hat{H}, \hat{Q}] \neq 0$ , we need to compute  $\hat{\mathcal{H}}(t)|\psi(t)\rangle$ . Multiplying by  $e^{i\hat{H}t}$ , we get  $e^{i\hat{H}t}\hat{\mathcal{H}}(t)e^{-i\hat{H}t}|\psi_0\rangle$ . Expanding  $e^{\pm i\hat{H}t}$  in power series and using Eq. (3), we get

$$\sum_{n=1}^{\infty} \frac{(a_0 n + a_0 + 1) i^n t^{n+1}}{(n+1)!} \underbrace{[\hat{H}, [\hat{H}, \dots [\hat{H}, \hat{Q}] \dots]]}_{n \text{ commutators}} |\psi_0\rangle. \quad (7)$$

This sum vanishes for arbitrary  $t$  only if each term vanishes independently.

(a) *Noninteracting spinless fermions.* To understand the structure of higher-order commutators in Eq. (7), it is useful to evaluate a few terms following the quadratic one:  $-(i/2)[\hat{H}, \hat{Q}]t^2 = -(\hat{n}_{-N} - \hat{n}_N)t^2$ , which acting on  $|\psi_0\rangle$  yields  $-t^2|\psi_0\rangle$ . The

prefactor of  $t^3$  is proportional to

$$[\hat{H}, (\hat{n}_{-N} - \hat{n}_N)] = i(\hat{j}_{-N}^{(1)} + \hat{j}_{N-1}^{(1)}), \quad (8)$$

where we have defined a generalized local current operator with support on  $m+1$  sites as:  $\hat{j}_l^{(m)} = (i\hat{f}_{l+m}^\dagger \hat{f}_l + \text{h.c.})$ . Analogously, one can define a generalized local kinetic energy operator:  $\hat{h}_l^{(m)} = (\hat{f}_{l+m}^\dagger \hat{f}_l + \text{h.c.})$ . In Eq. (8), the local current operators have support on two sites at the edges of the chain. Hence, they yield zero when applied to our initial state.

New terms in higher orders of  $t$  are generated symmetrically at both ends of the chain, so let us focus on the left end. The prefactor of  $t^4$  in Eq. (7) contains

$$[\hat{H}, \hat{j}_{-N}^{(1)}] = -i[\hat{h}_{-N}^{(2)} + 2(\hat{n}_{-N} - \hat{n}_{-N+1})]. \quad (9)$$

Again, this term vanishes when acting on the initial state.

Moving forward, the following commutation relations are useful:

$$[\hat{H}, \hat{j}_l^{(m)}] = -i\left[(\hat{h}_l^{(m+1)} - \hat{h}_{l-1}^{(m+1)}) + (\hat{h}_l^{(m-1)} - \hat{h}_{l+1}^{(m-1)})\right], \quad (10)$$

for the generalized local current operator,

$$[\hat{H}, \hat{h}_l^{(m)}] = i\left[(\hat{j}_l^{(m+1)} - \hat{j}_{l-1}^{(m+1)}) + (\hat{j}_l^{(m-1)} - \hat{j}_{l+1}^{(m-1)})\right], \quad (11)$$

for the generalized local kinetic energy operator, and

$$[\hat{H}, \hat{n}_l] = i(\hat{j}_l^{(1)} - \hat{j}_{l-1}^{(1)}), \quad (12)$$

for the site occupation operator. In Eqs. (10)–(12), the operator at site  $l-1$  vanishes if  $l = -N$ .

The central observation from Eqs. (10)–(12) is that the support of the local operators generated by the commutators grows linearly with the power of  $t$  in Eq. (7). The prefactor of  $t^m$  includes terms that are either of the form  $(\hat{h}_{-N}^{(m-2)} + \dots)$  or  $(\hat{j}_{-N}^{(m-2)} + \dots)$ . However, when acting on the initial state, those operators yield zero unless  $m-2 = N$  (i.e., their support extends over  $N+1$  sites). A second important observation is that the site occupations are nonzero if  $l < 0$ . From Eq. (10), for  $m = 1$ , one can see that site occupation operators  $\hat{n}_l$  and  $\hat{n}_{l+1}$  emerge in pairs. When acting on the initial state, those pairs cancel each other. This means that the first nonvanishing term in Eq. (7) (after  $t^2$ ) is



$[(N+1)t^{N+2}/(N+2)!] \times \mathcal{O}(1)$ . Using Stirling's formula ( $N \gg 1$ ) we see that this, and higher order terms, will be exponentially small if  $t \lesssim N/e$ . The numerical results in Fig. 1g confirm that this is indeed the case. Hence, by changing  $\lambda \rightarrow \lambda(t) = \lambda - t^2$ , we get that  $\hat{\mathcal{H}}(t)|\psi(t)\rangle$  is exponentially close to zero for  $t \lesssim N/e$ .

(b) *Spinless fermions with nearest-neighbor interaction.* Similar to noninteracting fermions, the operator  $\hat{Q}(V)$  is not conserved on a finite system with open boundaries since

$$\begin{aligned} [\hat{H}_V, \hat{Q}(V)] = & -i \left[ (1 + V^2/4) \left( \hat{h}_{-N+1}^{(1)} - \hat{h}_{N-1}^{(1)} \right) \right. \\ & + V \left( \hat{n}_{-N+1} - \hat{n}_{-N+2} \right)^2 \\ & \left. - V \left( \hat{n}_{N-1} - \hat{n}_N \right)^2 \right], \end{aligned} \quad (13)$$

with  $-N+1$  being the leftmost site. When acting on the initial state, this yields zero. Even though obtaining a general expression for the higher-order commutators is a daunting task, two consecutive higher order commutators (see Supplementary Information) confirm that the support of the off-diagonal operators grows linearly with the power of  $t$  in Eq. (7), and that site occupation operators emerge in pairs. Those commutators then vanish when acting on the initial state. As for  $V=0$ , we then expect the times for which the emergent Hamiltonian description is exponentially accurate to increase nearly linearly with  $N$ . The numerical results in Fig. 3a confirm this hypothesis.

## D. Computational details.

(a) *Infinite-repulsion Fermi- and Bose-Hubbard model.* We calculate the time-evolved wavefunction  $|\psi(t)\rangle$  and the ground state  $|\Psi_t\rangle$  of the emergent Hamiltonian in Eq. (5) using Slater determinants constructed from single-particle eigenstates that are the solutions to the spinless fermion Hamiltonian. For bosons and fermions at infinite repulsion, one

needs to express all operators in terms of spinless fermions operators. For impenetrable bosons, this transformation (due to Jordan and Wigner) is well established [25]. For the Fermi-Hubbard model at infinite onsite repulsion, we express one-particle operators for a given spin species as

$$\hat{f}_{l,\sigma}^\dagger \hat{f}_{l+x,\sigma} = \hat{f}_l^\dagger \hat{f}_{l+x} \hat{\mathcal{P}}_{l,x}^\sigma, \quad (14)$$

where  $\hat{\mathcal{P}}$  is a projector operator.

In one dimension, a spin-ordered initial state will preserve its ordering at all times (because of the constraint mentioned before). For the initial state of interest, the Néel state (see Fig. 1a),  $\hat{\mathcal{P}}$  is defined as

$$\begin{aligned} \hat{\mathcal{P}}_{l,x}^\sigma = & \prod_{j=l+1}^{l+x-1} \left( 1 - \hat{f}_j^\dagger \hat{f}_j \right) \times \\ & \frac{1}{2} \left( 1 + (-1)^{s_\sigma} e^{-i\pi \sum_{j=l+x+1}^N \hat{f}_j^\dagger \hat{f}_j} \right), \end{aligned} \quad (15)$$

where  $s_\downarrow = 0$  and  $s_\uparrow = 1$ . To calculate the expectation values of operators, both for impenetrable bosons and infinite-repulsion two-component fermions, we use the numerical procedure described in Ref. 15.

(b) *Spinless fermions with nearest-neighbor interaction.* We use full exact diagonalization to calculate the time-evolved wavefunction  $|\psi^V(t)\rangle$  and the eigenstate  $|\Psi_t^V\rangle$  of the emergent Hamiltonian  $\hat{\mathcal{H}}_V(t)$ . In  $\hat{\mathcal{H}}_V(t)$  (for the initial state studied in Fig. 3), we add a small onsite potential in the leftmost site ( $-10^{-3} \hat{n}_{-N+1}$ ) to break the degeneracy of  $|\Psi_t^V\rangle$  with other eigenstates with zero eigenenergy. This does not change  $|\Psi_t^V\rangle$ . It only changes its eigenenergy as  $\langle \hat{n}_{-N+1} \rangle = 1$  (maximal site occupancy) for the times for which the emergent Hamiltonian description is valid.

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## Supplementary Information

### EXPLICIT CHECK THAT THE TIME-EVOLVING STATE IS AN EIGENSTATE OF THE EMERGENT HAMILTONIAN

We consider the melting of a semi-infinite domain wall of infinitely repulsive spinful fermions. For this setup, the current ( $\hat{Q}$ , defined below) is exactly conserved, i.e.,  $[\hat{H}, \hat{Q}] = 0$ . We check that the state obtained by time evolving the initial domain wall is an eigenstate of the emergent Hamiltonian. We do this for the Fermi-Hubbard model with infinite onsite repulsion. The time-evolving state is obtained via the Bethe Ansatz using an integral representation for the initial state [31, 32]. All states are written assuming the infinite volume limit, and a complete set of states is used to represent the initial state,

$$|\psi_0\rangle = \int_{-\pi}^{\pi} \prod_j \frac{dk_j}{2\pi} A(\{k_j\}, \psi_0) |\{k_j\}\rangle, \quad (\text{S1})$$

$|\{k_j\}\rangle$  being an eigenstate of the model obtained via the Bethe Ansatz [33], and the  $A(\{k_j\}, \psi_0)$  are overlaps of the initial state with the eigenstates. We start with an initial Fock state,

$$|\psi_0\rangle = \prod_j \hat{f}_{x_j^0 \sigma_j^0}^\dagger |0\rangle \quad (\text{S2})$$

where the  $\sigma_j^0$  are the initial spins. Without loss of generality, we can assume that  $x_1^0 > x_2^0 > \dots > x_N^0$ . For this initial state, the overlaps are given by

$$A(\{k_j\}, \psi_0) = \sum_{\{x_j\}} \theta(x_1 > x_2 > \dots > x_N) \sum_P (-1)^P \prod_j e^{ik_{P_j} x_j - ik_j x_j^0} \quad (\text{S3})$$

where  $(-1)^P$  denotes the signature of the permutation. The sum over permutations of the product of single particle wave functions is essentially a Slater determinant. We time evolve this state by direct application of the time-evolution operator, which acts on the eigenkets to produce phase factors  $\prod_j e^{-iE(k_j)t}$  with  $E(k_j) = -2 \cos k_j$  (the hopping parameter  $J = 1$ ). Using this procedure, we obtain a time-evolving state given by

$$|\psi(t)\rangle = \sum_{\{x_j\}} \int \prod_j \frac{dk_j}{2\pi} \theta(x_1 > x_2 > \dots > x_N) \sum_P (-1)^P \prod_j e^{2it \cos k_j + ik_{P_j} x_j - ik_j x_j^0} \hat{f}_{x_j \sigma_j^0}^\dagger |0\rangle. \quad (\text{S4})$$

This expression is easy to interpret. The anti-symmetrization imposed on the wave function guarantees the fermionic symmetry and the explicit ordering via the  $\theta$ -function prevents spin exchange.

We then explicitly evaluate the action of the emergent Hamiltonian on the time-evolving state  $|\psi(t)\rangle$ . For this model, the emergent Hamiltonian is given by

$$\hat{\mathcal{H}}(t) = \hat{\mathbf{S}} \left[ \sum_{j,\sigma} j \hat{f}_{j,\sigma}^\dagger \hat{f}_{j,\sigma} - it \sum_{j,\sigma} \left( \hat{f}_{j+1,\sigma}^\dagger \hat{f}_{j,\sigma} - \hat{f}_{j,\sigma}^\dagger \hat{f}_{j+1,\sigma} \right) \right] \hat{\mathbf{S}} - \sum_j x_j^0 \quad (\text{S5})$$

The projection operator  $\hat{\mathbf{S}}$  projects any state into the subspace without doubly occupied sites. It reproduces the effect of the infinitely repulsive interaction, and has to be retained in the emergent Hamiltonian. It can be represented by  $\hat{\mathbf{S}} = \prod_j (1 - \hat{n}_{j,\uparrow} \hat{n}_{j,\downarrow})$ . The rest of the above equation is similar to Eq. (5) in the main text: the first term in the brackets is the initial Hamiltonian  $\hat{P}$ , the second term is  $a_0 t \hat{Q}$ , and the last term (outside the brackets) is  $\lambda$ . The action of the second term in the above equation (call it  $\hat{\mathcal{H}}_c$ ) gives

$$\hat{\mathcal{H}}_c |\psi(t)\rangle = \sum_{x_j} \sum_m \theta(x_1 > x_2 > \dots) \int \prod_j \frac{dk_j}{2\pi} \prod_j e^{2it \cos k_j + ik_j(x_j - x_j^0)} (-2t \sin k_m) \prod_j \hat{f}_{x_j \sigma_j^0}^\dagger |0\rangle, \quad (\text{S6})$$

The first term on the other hand, gives

$$\hat{\mathcal{H}}_p |\psi(t)\rangle = \sum_{x_j} \sum_m \int \theta(x_1 > x_2 > \dots) \prod_j \frac{dk_j}{2\pi} \prod_j e^{2it \cos k_j + ik_j(x_j - x_j^0)} x_m \prod_j \hat{f}_{x_j \sigma_j^0}^\dagger |0\rangle. \quad (\text{S7})$$

Integrating this by parts, adding to the second term and putting back the third term, we get

$$\hat{\mathcal{H}}(t) |\psi(t)\rangle = 0, \quad (\text{S8})$$

proving our claim explicitly. The proof for impenetrable bosons works similarly, with the emergent Hamiltonian given by Eq. (5) in the main text.

## EMERGENT LOCAL HAMILTONIAN FOR INTERACTING SPINLESS FERMIONS

Here we continue our analysis of the validity of the emergent Hamiltonian description for spinless fermions with nearest-neighbor interaction. We have argued in the Methods that the energy current operator  $\hat{Q}(V)$  is not exactly conserved on a finite lattice with open boundaries, and provided the expression for  $[\hat{H}_V, \hat{Q}(V)]$ . The second-order commutator is given by:

$$\begin{aligned} [\hat{H}_V, [\hat{H}_V, \hat{Q}(V)]] \Big|_{\text{left boundary}} &= \\ \left\{ \hat{j}_1^{(2)} - V \left( \hat{j}_1^{(1)} \left( \hat{n}_3 - \frac{1}{2} \right) + 2\hat{j}_2^{(1)} \left( \hat{n}_1 - \frac{1}{2} \right) \right) + \frac{V^2}{4} \hat{j}_1^{(2)} - \frac{V^3}{4} \hat{j}_1^{(1)} \left( \hat{n}_3 - \frac{1}{2} \right) \right\}. \end{aligned} \quad (\text{S9})$$

The maximal support of the local operators extends on the three leftmost lattice sites, and gives zero when acting on the initial state. In the same manner, the third-order commutator acts at most on the four leftmost lattice sites,

$$\begin{aligned} [\hat{H}_V, [\hat{H}_V, [\hat{H}_V, \hat{Q}(V)]]] \Big|_{\text{left boundary}} &= i \left[ - \left( \hat{h}_1^{(3)} + (\hat{h}_1^{(1)} - \hat{h}_2^{(1)}) \right) \right. \\ &+ V \left\{ \hat{h}_1^{(2)} \left( \hat{n}_4 - \frac{1}{2} \right) + 2\hat{h}_2^{(2)} \left( \hat{n}_1 - \frac{1}{2} \right) - \hat{h}_1^{(2)} \left( \hat{n}_2 - \frac{1}{2} \right) - \hat{j}_1^{(1)} \hat{j}_3^{(1)} \right. \\ &\quad \left. \left. + 2(\hat{n}_1 - \hat{n}_2) \left( \hat{n}_3 - \frac{1}{2} \right) + 2(\hat{n}_2 - \hat{n}_3) \right\} \right. \\ &- \frac{V^2}{4} \left\{ \hat{h}_1^{(3)} + (\hat{h}_1^{(2)} - \hat{h}_2^{(1)}) + 4\hat{h}_1^{(1)} \left( \hat{n}_3 - \frac{1}{2} \right) + 8\hat{h}_2^{(1)} \left( \hat{n}_1 - \frac{1}{2} \right) \left( \hat{n}_4 - \frac{1}{2} \right) - 2\hat{h}_2^{(1)} \right\} \\ &+ \frac{V^3}{4} \left\{ \hat{h}_1^{(2)} \left( \hat{n}_4 - \frac{1}{2} \right) + \hat{h}_1^{(2)} \left( \hat{n}_2 - \frac{1}{2} \right) - \hat{j}_1^{(1)} \hat{j}_3^{(1)} + 2(\hat{n}_1 - \hat{n}_2) \left( \hat{n}_3 - \frac{1}{2} \right) \right\} \\ &\left. - \frac{V^4}{4} \hat{h}_1^{(1)} \left( \hat{n}_3 - \frac{1}{2} \right) \right]. \end{aligned} \quad (\text{S10})$$

Locality of the Hamiltonian  $\hat{H}_V$  guaranties that in every new generation, the support of operators will increase at most for one site, similarly to noninteracting particles. This implies that the  $N$ -th order commutator, having support on  $N + 1$  sites, will be the first operator to yield a nonzero value when acting on the initial state. For lower orders, the only terms that do not vanish are the density operators. However, they emerge in pairs and cancel each other after acting on the initial state. Our numerical results show that the target eigenvalue equals zero for all times when the emergent Hamiltonian description is valid, which suggests that, at least for commutators up to the  $N$ -th order, the density operators cancel each other after acting on the initial state.